Shear-Layer-Edge Interaction: Simulation by Finite-Area Vortices

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Coherent vortices have been observed within impinging shear layers under certain conditions. It is believed that the feedback force necessary for the generation of these vortices is induced by their interaction with the impingement surface. In the present work, the shear-layer/surface interaction is simulated by considering the impingement of a row of finite-area vortices on an edge. The proposed model simulates efficiently the shear-layer/edge interaction. In addition, these results provide solid support of the hypothesis that the pressure waves that are emitted from an impingement edge are generated by the vortices/edge interaction. Appropriate parametric application of the present method indicates that the amplitude of the pressure waves depends strongly on the length of the succession of the vortices and on the frequency of their release. This amplitude becomes smaller when the spacing of the vortices decreases while the succession length remains constant, or when the succession length decreases while the number of the vortices remains constant. These conclusions are useful in explaining the following features of a self-oscillating impinging shear layer: when the length of the shear layer is constant, there is a specific limit of the maximum mode of oscillation that can be established, whereas for increasing shear-layer length, a critical value is reached above which the next mode appears.

I. Introduction

THE self-sustained oscillations of an impinging shear layer is a well-known phenomenon that appears to have a variety of applications, such as about slots between the moving parts of control surfaces of aeroplanes, hydraulic gates, and spiked cones of re-entry vehicles. Experimentally, it has been found that, in such a flow, the shear layer that impinges on a surface may oscillate periodically. This oscillation leads to the emission of strong acoustic radiation, to an increase of the drag and heat transfer (in the case of high speeds), and, possibly, to vibrations of the local structure.

An early description of the feedback cycle leading to the establishment of self-sustained oscillations of impinging shear flows has been given by Rayleigh.[1] Furthermore, the classical pictures of Brown[2] revealed that in an edge-tone system vortices are shed periodically near the separation point and travel downstream toward the edge. Periodic vortices also were detected in a cavity flow by Rossiter,[3] who speculated that they are shed at the upstream corner in sympathy with the pressure oscillation produced by interaction of the vortices with the downstream corner.

Contemporary views are well summarized by Rockwell[4]: "For most of these oscillations to be self-sustained, a chain of events must occur: impingement of organized vorticity fluctuations upon the edge/surface; resultant upstream influence (interpreted as Biot-Savart induction or upstream pressure waves); conversion of disturbances incident upon the region of the shear layer in the vicinity of the separation edge to velocity fluctuations within the shear layer; and amplification of these fluctuations in the streamwise direction." Visualization pictures taken from the papers of Ziada and Rockwell[5] and of Rockwell and Knisely[6] are shown in Fig. 1 for an edge-tone and for a cavity flow.

Concerning the theoretical investigation of the conversion of the pressure perturbations at the shear-layer separation to vorticity fluctuations within the shear layer, little has been done, to our knowledge. Rockwell and Naudascher[7] mention that, according to an experimental analysis of Morkovin, the back and forth motion of the flow detachment point allows the transformation from irrotational pressure to rotational vorticity perturbations. Besides, Durbin,[8] who models the shear layer by a thin vortex sheet, has shown that if, in addition to the condition of impermeability of the surface, the Kutta condition is applied at the sharp separation point, the unsteady linear disturbance equations have solutions only for certain pairs of frequency and distance between the edge. Thus, by including the Kutta condition, the feedback between downstream and upstream edges may be analyzed.

In many theoretical studies, this element of the feedback mechanism is modeled by its result: the periodic shedding (or release) of vortices from the origin of the shear layer. Then, various characteristics of the feedback cycle can be studied. Typical examples of application of this technique are those of Curle[9] and of Rossiter,[3] who were able to derive formulas for the frequency of oscillation and explain the jumps it presents by simulating the shear layers by successions of point vortices released from the origin of an edge-tone system or from the upstream lip of a cavity, correspondingly. In these models, the succession of the vortices is "frozen." However, for the study of the other basic element of the feedback mechanism, i.e., the generation of the periodic pressure disturbances at the reattachment edge, the kinematics of the vortices and their dynamic effects must be considered.

For the task just described, appropriate modeling of the flow is necessary. The replacement of the finite-size vortices by single-line (discrete) vortices is an approximation that has been applied successfully in many vortex/surface interactions. Then the techniques of complex variables allow the calculation of the trajectories of the discrete vortices. These trajectories are qualitatively similar to those of the centroids of the finite-size vortices (Saffman and Baker[10]), unless the vortices pass very close or impinge on the surface (Rockwell and Knisely[6]). Some typical examples of application of this method are reviewed by Saffman and Baker.[10]

For the specific case of the impinging shear layers, Conlisk and Rockwell[11] were the first to apply this technique for the successful calculation of the pressure fluctuations induced on

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Fig. 1 Experimental examples of vortex/edge interactions (courtesy of Prof. D. Rockwell).

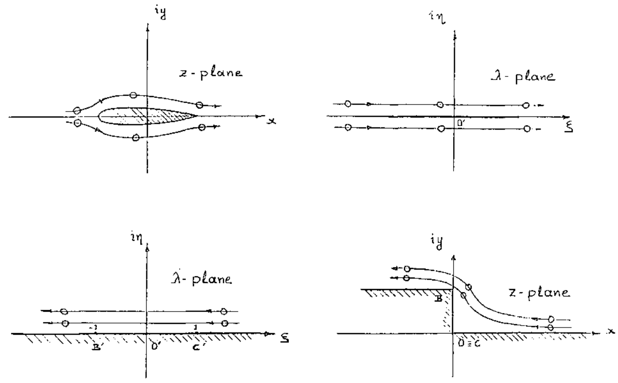

Fig. 2 Models of vortices/edge interaction.

a corner by a single discrete vortex or by patterns of vortices similar to those observed experimentally. Also, the present author applied this technique to study the effect of the geometry of the reattachment edge on the amplitude of the pressure pulses induced by a single discrete vortex (Panaras[12]). He examined two typical shapes of the edge: a ramp of variable angle and an ellipse. He found that the amplitude of the pulses depends strongly on the specific shape of the edge. Thus, pressure pulses on reattachment edges with shape that is known, experimentally, to result in steady flows (i.e., rounded edges or ramps of small angle) have insignificant amplitude. On the contrary, when the reattachment edge has a shape that is known to result in oscillating flow (ramps of large angle), the induced pressure pulses are of very large amplitude. This comparison indicates that for the establishment of sustained oscillations in a cavity, the existence of a periodic feedback force greater than a minimum value is necessary.

A similar conclusion has been reached by Gharib[13] while studying experimentally the receptivity of a cavity shear layer to acoustic forcing, for a cavity length less than the one required for the onset of self-sustained oscillations. He has found that the resonance appears when the forcing frequency satisfies the phase criterion and when the forcing level reaches a threshold level, which depends on the length of the cavity.

The necessity of considering the finite area of the vortices in a vortex/surface interaction has been addressed recently by the present author (Panaras[14]). Simulating the vortex/airfoil interaction by both techniques, he found that, for relatively large distances of the interacting vortex from the surface of the airfoil, the single-point-vortex technique provides results similar to those of the finite-area method. However, when the distance of the vortex from the surface of the airfoil is small, its shape is distorted, and the induced pressure pulses have smaller amplitude than the ones induced by an equivalent line vortex. In the limit, where the vortex impinges on the leading edge of the vortex, it is split in two, and the time-dependent pressure pulses even take negative values at some parts of their period.

In this work, a model that simulates efficiently the types of flows shown in Fig. 1 will be developed. In this model, the vortical structures will be simulated by arrays of discrete vortices. Thus, it will be possible to study impingement and splitting of vortices on an edge. The model will be used to study the dependence of the shape and the amplitude of the pressure pulses induced on an edge on some primary features of the impinging succession of vortices.

II. Description of the Model

This section describes the proposed model for the simulation of interactions similar to those shown in Fig. 1, where the interacting shear layer is transformed into well-developed vortices. According to the experimental evidence, when a vortex approaches an edge or a corner, it may pass above the surface, or it may inpinge on it and be split in two. Modeling the latter possibility is not easy by any step or block transformation that can be used for the simulation of a cavity-type flow because whereas in a real cavity flow (Fig. 1) the vortices pass above a "dead air" region, in a numerical simulation the vortices will be embedded within a parallel stream that extends down to the horizontal axis. Hence, even if the simulated succession of vortices initially lies very near this axis, still no splitting of the vortices will happen (Fig. 2b). On the contrary, if an edge is used as the impingement surface of the model, splitting the vortices is possible because any element of a vortex that lies below the horizontal axes will pass below the edge (Fig. 2a). For the aforementioned reasons, an edge will be used in the present model primarily as an impingement surface, while a step will be used in a limited number of applications for demonstrating the global character of the mechanism that induces the feedback pressure waves.

The appropriate modeling of the Kutta condition, which requires finite values of the pressure in the vicinity of the tip of a sharp edge, is another numerical difficulty. This may be accomplished by releasing additional vorticity at the tip, a procedure that is rather difficult in practice. Recently, Kaykayoglu and Rockwell,[15] based on their experimental results, according to which the pressure amplitude is maximum at the tip, suggest that inviscid modeling should not incorporate a leading-edge Kutta condition. Instead, a singularity at the tip would seem to be most representative of the real conditions. In the present work, the issue of the Kutta condition is overcome by selecting an edge of finite thickness as an impingement surface.

Concerning the modeling of the impinging vortices, an array of discrete vortices will be used for the simulation of their finite area. Initially, each vortex will be represented by a disturbed vortex sheet of finite thickness composed of four rows of discrete vortices. Then, by application of the Biot-Savart law and by numerical integration in small time steps, the evolution of the array is estimated, leading to vorticity concentrations that have been observed in nature or the laboratory. In this technique, the source of the initial perturbation of the vortex array can be neither defined nor connected to it by means of a feedback cycle. The origin of this technique goes back to Rosenhead[16] and Acton,[17] who studied the stability of a semi-infinite vortex sheet. As has been mentioned, the present author extended this technique to the study of a vortex/airfoil interaction (Panaras[14]).

The modeling of the secondary vortices, which may be formed along the edge if the primary vortices are strong enough to separate the boundary layer, is another critical issue. It is very difficult to treat this secondary shedding numerically as a truly self-generated phenomenon (Rockwell[4]). This feature of the real flows will not be included in the present model. Considering the fact that when a secondary vortex appears, it is nested within the primary vortex (Rockwell et al.[18]), it is expected that the impact of this approximation will not be significant in the present case, where the induced pressure fluctuations will be estimated only in one point.

The method of complex transformations will be used in the present work. The curve that defines the geometry of any particular edge in the z plane will be transformed by means of a transformation function $z=f(\lambda)$ into a segment of the horizontal axis in the transformed λ plane. Then it is easy to estimate the velocity potential. For the application of the method, the simulated vortical structures will be released periodically from the "origin" of the flow at a predetermined frequency. If each vortical structure is composed of N discrete vortices, each of Γ strength, the complex velocity potential at a point λ in the transformed plane is

$$F(\lambda)=U_\infty\lambda+\frac{i\Gamma}{2\pi}\sum_{n=1}^{N}\ln(\lambda-\lambda_n)-\frac{i\Gamma}{2\pi}\sum_{n=1}^{N}\ln(\lambda-\bar{\lambda}_n) \qquad (1)$$

The velocity field induced on a point z in the physical plane is given by

$$u(z)-iv(z)=\frac{\mathrm{d}F}{\mathrm{d}\lambda}\frac{1}{f'(\lambda)}=\left[1+\frac{iK}{2\pi}\sum_{n=1}^{N}\frac{1}{\lambda-\lambda_n}-\frac{iK}{2\pi}\sum_{n=1}^{N}\frac{1}{\lambda-\bar{\lambda}_n}\right]\frac{1}{f'(\lambda)} \qquad (2)$$

In Eq. (2), the velocities have been nondimensionalized on U_∞, the lengths on an appropriate length a, and $K=\Gamma/aU_\infty$.

For the calculation of the velocity of a vortex located at a point z_j, Routh's rule must be used, leading to

$$u_j-iv_j=\left[1+\frac{iK}{2\pi}\sum_{\substack{n=1\\n\neq j}}^{N}\frac{1}{\lambda_j-\lambda_n}-\frac{iK}{2\pi}\sum_{n=1}^{N}\frac{1}{\lambda_j-\bar{\lambda}_n}\right]\frac{1}{f'(\lambda_j)}-\frac{iK}{4\pi}\frac{f''(\lambda_j)}{[f'(\lambda_j)]^2} \qquad (3)$$

The trajectory of any vortex in the flow will be estimated by solving numerically, at successive time steps Δt, the equations

$$\frac{\mathrm{d}x_j}{\mathrm{d}t}=u_j(x_j,y_j) \qquad (4a)$$

$$\frac{\mathrm{d}y_j}{\mathrm{d}t}=v_j(x_j,y_j) \qquad (4b)$$

Since the velocity components u_j, v_j are given in terms of the variable λ_j, inversion of the transformation $z=f(\lambda)$ is necessary. In the case of the edge, this inversion is straightforward, but in the case of the corner, a numerical solution will be applied.

The periodic pressure fluctuations induced on a point z of an edge by a succession of vortices may be estimated by the equation

$$\rho\frac{\partial\phi}{\partial t}+p_k+\frac{1}{2}\rho(u_k^2+v_k^2)=p_\infty+\frac{1}{2}\rho U_\infty^2 \qquad (5)$$

where index k indicates that the local velocity at the point z includes the effect of the vortices in addition to the mean flow contribution [Eq. (2)] and the term $\rho\partial\phi/\partial t$ denotes the nondimensional unsteady potential.

If from Eq. (5) the value of the pressure coefficient caused only by the mean flow is subtracted (velocity components u,v), the following coefficient is derived:

$$c_p=(u^2+v^2)-(u_k^2+v_k^2)-\frac{\rho}{q_\infty}\frac{\partial\phi}{\partial t} \qquad (6)$$

This coefficient has the advantage of referencing the pressure (positive values) or suction (negative values) induced by the convected array of vortices on the local value of the mean-flow-caused pressure. The nondimensional unsteady potential term is found to be given by the equation

$$\frac{\rho}{q_\infty}\frac{\partial\phi}{\partial t}=\frac{K}{\pi\Delta t}\sum_{n=1}^{N}\Delta[\arg(\lambda-\bar{\lambda}_n)-\arg(\lambda-\lambda_n)] \qquad (7)$$

where Δt is the nondimensional time step used for the numerical integration of the kinematic equations.

III. Interaction with a Leading Edge of Finite Thickness

A symmetrical Joukofski airfoil will be used for studying the vortices/edge interaction. The following successive transformations transform the flow about a symmetrical airfoil into the flow about a line segment on the λ plane (Fig. 3):

$$z=(g+b)+\frac{1^2}{(g+b)} \qquad (8)$$

$$\lambda=g+\frac{a^2}{g} \qquad (9)$$

where 1 and b are parameters that define a particular airfoil and a is the radius of the basic circle into which the airfoil is transformed at the g plane. The radius a will be used for the nondimensionalization of the various length parameters. The particular airfoil used in this work is defined by the parameters $1=0.9$, $b=0.1$.

The main parameters for the application of the present model are the distance L between the point of release of the distributed vorticity structures and the edge, the spacing λ of the released vortices, and the vertical offset of the edge relative to the centroids of the vortices. The distance L simulates the length of the shear layer before the impingement. In all the cases, the vortices will be released one by one from the initial point ($-L$, offset), with a spacing corresponding to the relation $\lambda=L/n$ where n is the mode number.

In the various figures, the length of the shear layer and its offset distance from the airfoil will be referenced to the chord of the airfoil. In most of the cases, a typical nondimensional

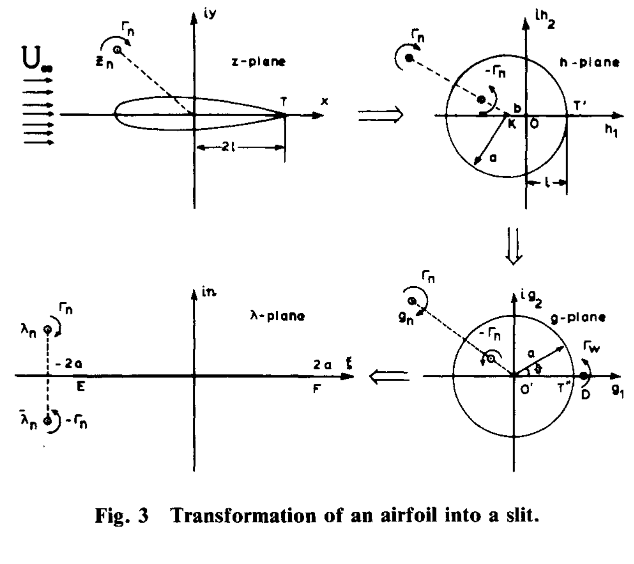

Fig. 3 Transformation of an airfoil into a slit.

length equal to $L=1.4$ will be used. Also, a standard value $K=0.45$ will be given to the nondimensional strength of the vortical structures. This particular value has been selected because in this case, at the time the released vorticity distributions reach the edge, their rolling up has been completed, and they have been transformed into regular vortices.

A. Effect of the Offset Distance

For examining the effect of the offset distance of the edge on the amplitude of the pressure pulses that are induced on it, three cases of interaction are shown in Fig. 4 for the following successive values: offset $= -0.13$, 0, 0.13. The mode number in this case is $n=2$. In Fig. 4a, the offset of the vortices is such that they just pass above the upper surface of the airfoil without touching it. In Fig. 4b, the centroids of the vortex structures lie on the horizontal axe, so they are split into two equal parts when they impinge on the leading edge of the airfoil. In Fig. 4c, the vortices pass below the edge.

Referring to Fig. 4a to explain the data shown in each figure, the wavy curve depicts the time-dependent pressure fluctuation at a point A of the leading edge of the airfoil. Part of the pressure fluctuation that corresponds to the time required for a released vortical structure to reach the leading edge of the airfoil is shown. Also, in each figure, the level of the zero-value of the pressure coefficient is given, plus the maximum amplitude of the pressure waves. In all the figures, a "picture" of the vortices is shown at the time the calculation of the pressure coefficient terminates. The mark of the origin of the flow in this and all the other figures is schematic; there is no upstream plate in the flow.

A review of the evolution of the vortical structures in the various cases shown in Fig. 4 shows that the present model simulates very efficiently the real phenomenon. The rolling up of the shear layer, the formation of the rounded vortices, their clockwise rotation, their deformation when they approach the edge, and their split if they impinge on the edge are quite similar to the corresponding features of the laboratory vortices shown in Fig. 1.

Concerning the pressure fluctuations, it is observed that when the vortices are not split (Fig. 4a), they have the shape of smooth pressure waves that follow the frequency of the vortices. When the vortices split (Fig. 4b), the pressure fluctuations are less smooth. Besides, it is noted in Fig. 4 that the split of the vortices also affects the level of the mean pressure, which gradually falls to zero. In the last case, where the vortices pass below the edge, the pressure fluctuations are negative. If these pressure waves are compared to those of Fig. 4a, they are seen to have higher amplitude and a phase difference equal to π. It is evident that the appearance of positive or negative pressure fluctuations depends on the direction of rotation of the vortices and on the relative position of the point A. For clockwise vorties examined here, if the point A lies above them, the induced velocity will have the direction of the parallel flow; thus, a suction will appear. On the contrary, if the point A lies below the vortices, a velocity opposite to the parallel flow will be induced, causing pressure waves to be emitted.

Booth,[19] using a symmetric airfoil, has shown experimentally that significant pressure fluctuations are induced only in the leading-edge area of an airfoil. This feature is predicted by the present model. Details are given in Sec. V. In addition, it is clarified that both the development of the vortical structures and the amplitude of the pressure fluctuation depend not only on the selected value of the total circulation of each vortical structure but also on the distribution of the point vortices. Bringing the point vortices closer or increasing their strength results in the formation of the finite-area vortices at a shorter distance from their origin.

It is interesting to examine the contribution of the unsteady potential to the time-dependent surface pressure. For this purpose, in Fig. 5, the two components of the pressure are shown for some cases studied in Fig. 4. The lines denoted by the

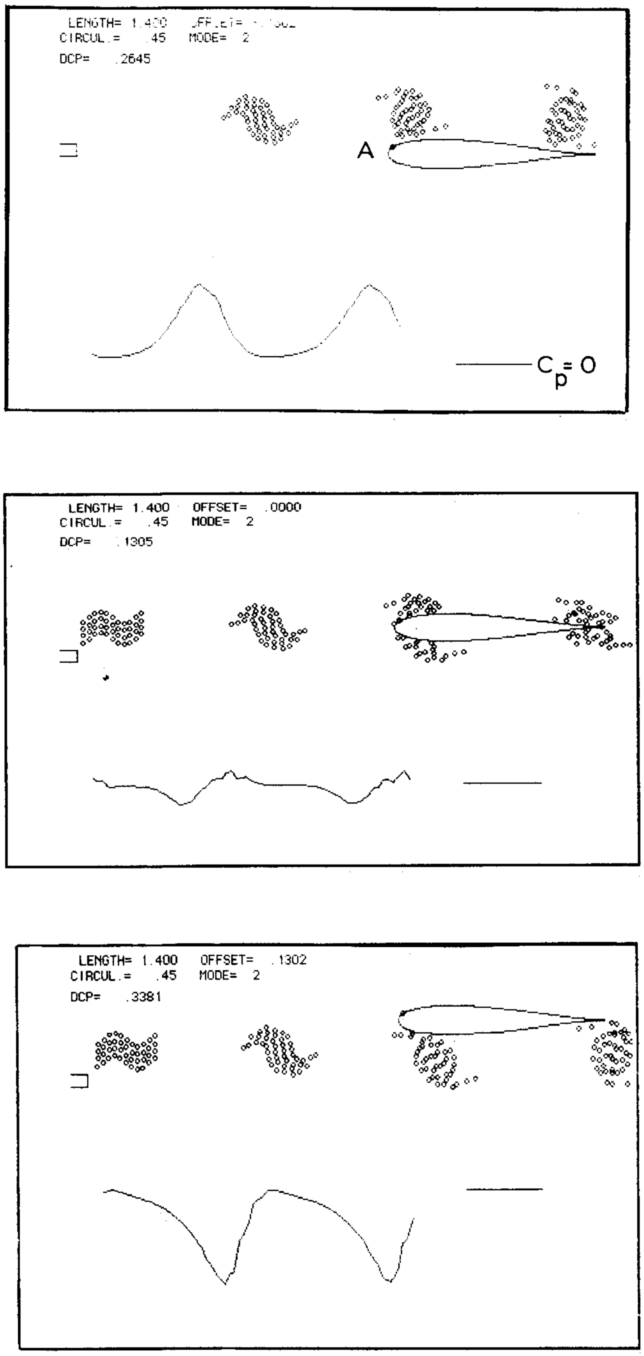

Fig. 4 Effect of the offset distance on the interaction.

number 1 correspond to the contribution of the potential field distortion because of the presence of the vortices [velocity components u_k and v_k in Eq. (6)]. We will call this "steady" flow contribution. The curves denoted by the number 2 correspond to the unsteady potential contribution. A review of the curves shows that both contributions are similar and in phase, the steady flow contribution having the greater amplitude. Thus, the unsteady potential tends to reduce the amplitude of the final pressure fluctuation. This tendency has been found in the case of the interaction of a vortex with a ramp (Panaras[12]). However, in the latter case, the amplitude of the unsteady term was much smaller than the amplitude of the steady term.

B. Effect of the Spacing of the Vortices

For studying the effect of the spacing of the vortices, in Fig. 6, the case of Fig. 4a is repeated, but for mode numbers $n=3$,

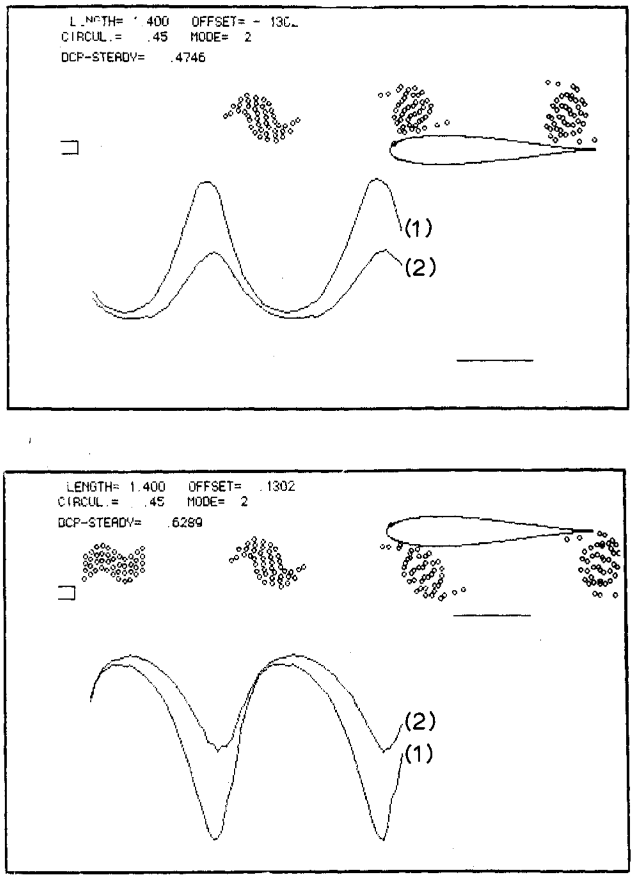

Fig. 5 Decomposition of the pressure fluctuation into the steady (curve 1) and the unsteady (curve 2) potential component.

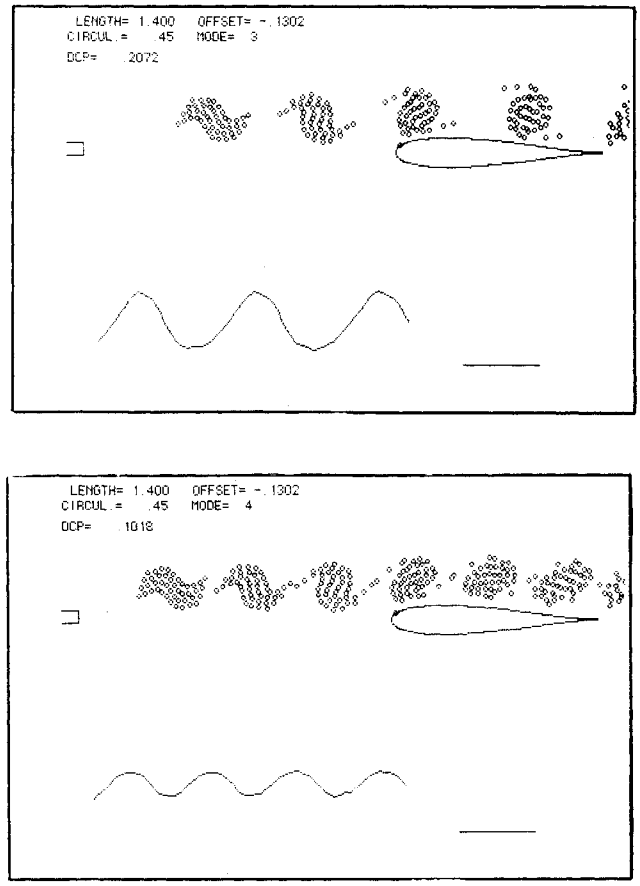

Fig. 6 Effect of the spacing of the vortices: flow above the edge.

4. Comparison of these figures shows that when the spacing is increased, the mutual interaction forces of the vortices become greater. This is indicated by the change of the shape of the vortices. Besides, as the spacing of the vortices decreases the amplitude of the pressure waves decreases, while the mean pressure increases. The observed decrease of the pressure amplitude for increasing values of the mode number is very rapid. Between the third and fourth modes, the maximum pressure amplitude is reduced to a half. The same behavior is observed when the vortices are symmetrically split (Fig. 7a) or when they pass below the edge (Fig. 7b).

The preceding results are probably related to the fact that in a self-oscillating shear layer, there is a specific limit of the maximum mode of oscillation that can be established for a constant length. It seems that this limit exists because, as the number of mode is increased, the amplitude of the feedback force induced on the downstream edge becomes less than the one required for the excitation of the shear layer to the level required for the generation of the large-scale vortices.

C. Effects of Other Parameters

The distance between the separation point of a mixing layer and the edge where it impinges is a basic parameter of the self-oscillating shear layers. For studying the effect of this length on the amplitude of the pressure waves induced on the edge, results of calculation are shown in Fig. 8 for a length $L = 0.7$ and the second mode. If these results are compared with the ones shown in Fig. 4 (for a length $L = 1.4$), it is seen that whether the vortices pass above or below the edge, the amplitude of the pressure waves becomes smaller when the shear-layer length is decreased. Increase in the amplitude is observed only when the vortices are split into two equal parts. If these results are extrapolated to the oscillating shear layers, they indicate, qualitatively, the following tendency: for the establishment of a specific mode of periodic oscillation, the shear-layer length should be greater than a minimum limit. The greater the mode number is, the greater the required shear-layer length becomes. This tendency is in agreement with the experimental evidence.

It is of practical importance to investigate the effect of jets or mixing layers to adjacent surfaces. For this, a case is shown with the offset distance equal to the thickness of the shear layer in Fig. 9. It is seen that when the spacing of the vortices is relatively large (Fig. 9a), a significant vertical elongation of their shape is observed at the time they have reached the region of the trailing edge of the airfoil. This elongation is due to the strain imposed on the vortices by the nonuniform flowfield. However, no elongation at all is observed when the vortices are closely spaced (Fig. 9b). Evidently, in this case, the effect of the local velocity field has been offset by the mutual interaction forces of the vortices. Concerning the pressure fluctuations, they are seen to have the shape of smooth waves. Their mean value is greater than zero, but their amplitude is very small, and it becomes almost insignificant when the spacing of the vortices is reduced. In view of these results, it seems that a surface adjacent to a mixing layer or a jet feels a constant pressure, but for the appearance of a fluctuating pressure, the vortices must pass very near the surface.

For the completeness of the analysis, some examples of calculation are shown in Fig. 10 with the vortices impinging on the sharp edge of the airfoil. It is observed that the evolution of the vortices and the variation of the pressure fluctuations at a point near the tip of the sharp edge are very similar to the case of the finite-thickness edge, the amplitude being higher in the latter case.

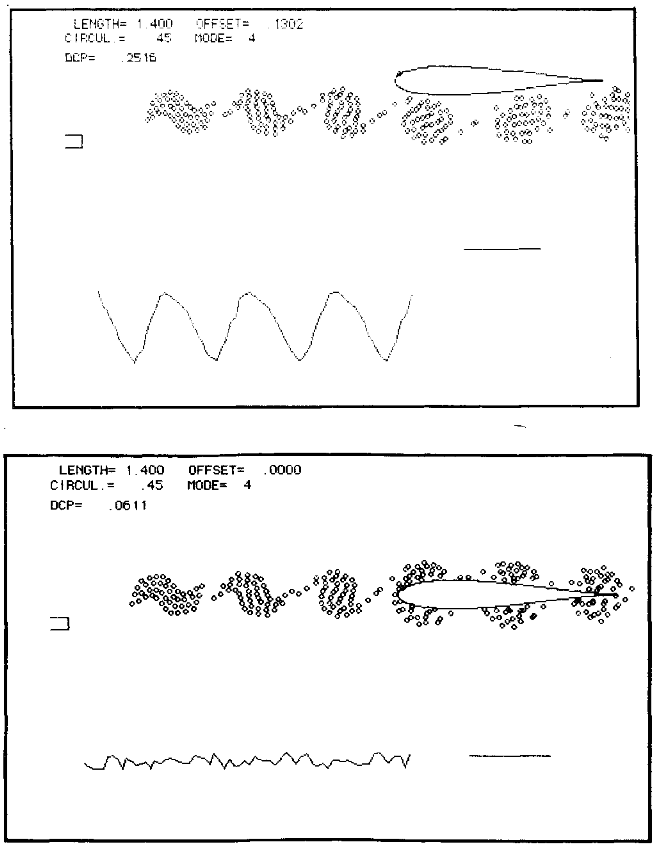

Fig. 7 Effect of the spacing of the vortices: other cases.

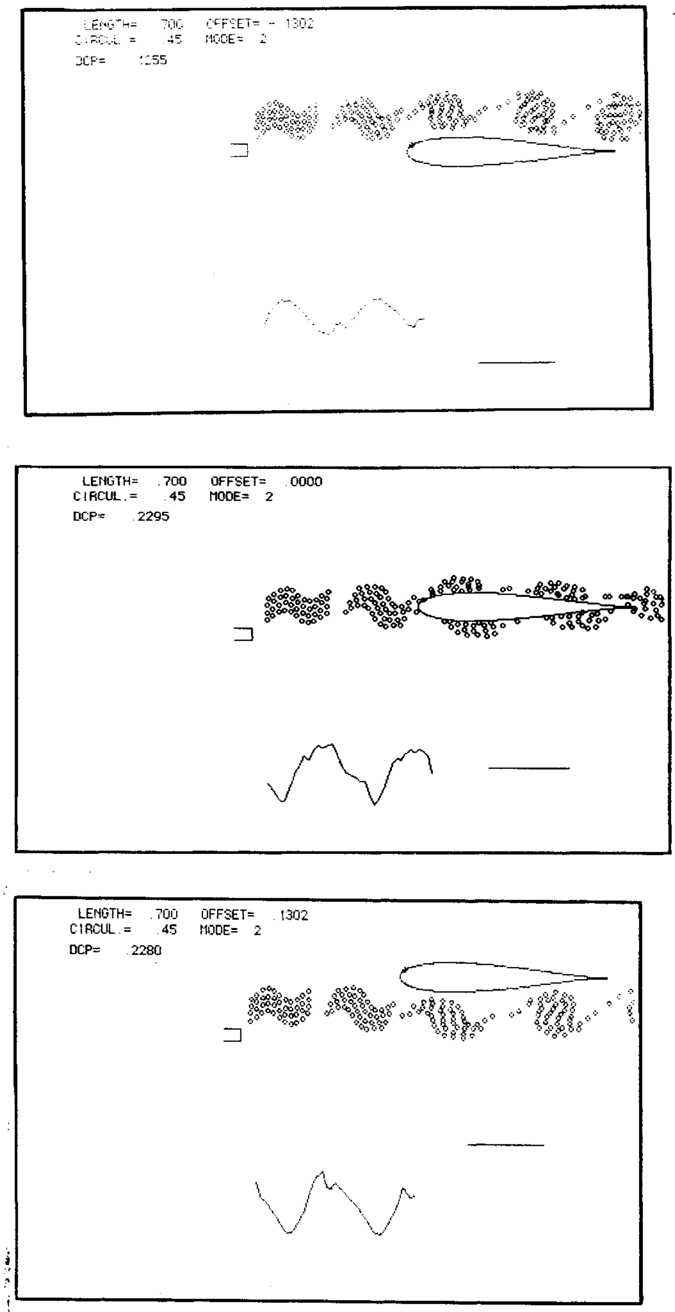

Fig. 8 Effect of the length of the succession of the vortices.

IV. Interaction with a Corner

For the simulation of a cavity-type flow, the following transformation that transforms a step into a line (Fig. 2b) will be used:

$$z = f(\lambda) = [\sqrt{(\lambda^2 - 1)} + \cos^{-1}\lambda]\,(h/\pi) \tag{10}$$

In order to match the velocity at infinite on the transformed plane to the corresponding one on the physical plane, the value $h = \pi$ is given to the step height. Also, for the nondimensionalization of the various lengths of the equations of Sec. II, the step height will be used.

The transformation equation, Eq. (10), has been selected because the separation of the variables, as well as the calculation of the first and of the second derivative, are easily done. As was mentioned in Sec. II, for the estimation of the induced velocity field it is necessary to invert the equations:

$$x = F(\xi,\eta) \tag{11a}$$

$$y = G(\xi,\eta) \tag{11b}$$

This inversion has been done numerically by applying the Newton method described by Conte and de Boor.[20]

As was mentioned in Sec. II, it is not possible to simulate numerically the experimentally observed tearing of a vortex when it impinges on a corner. Thus, in the present section, only the case of the convection of a succession of vortices above the corner will be examined. In Fig. 11, calculations similar to those of Figs. 4a and 6a are shown. A comparison shows that the pressure fluctuations induced on the corner are similar to those induced on an edge. Thus, in the corner flow also, smooth pressure waves are induced that follow the frequency of the vortices. Their amplitude decreases, while their mean value increases when the spacing of the vortices is decreased. Besides, comparison of the development of the vortices with the experimental ones shown in Fig. 1 indicates that in this case also the model simulates efficiently the real phenomenon.

Finally, for studying the effect of the length of the "cavity" on the mode number, an example is shown in Fig. 12 for a length $L = 3$ and the second mode. Again, the tendency is similar to that of the vortex/edge interaction case, i.e., the amplitude of the induced pressure waves is decreased if the length of the impinging row of vortices is decreased for the same mode number.

V. Discussion and Conclusions

The applications of the present model in the previous sections demonstrate its efficiency in simulating the basic features

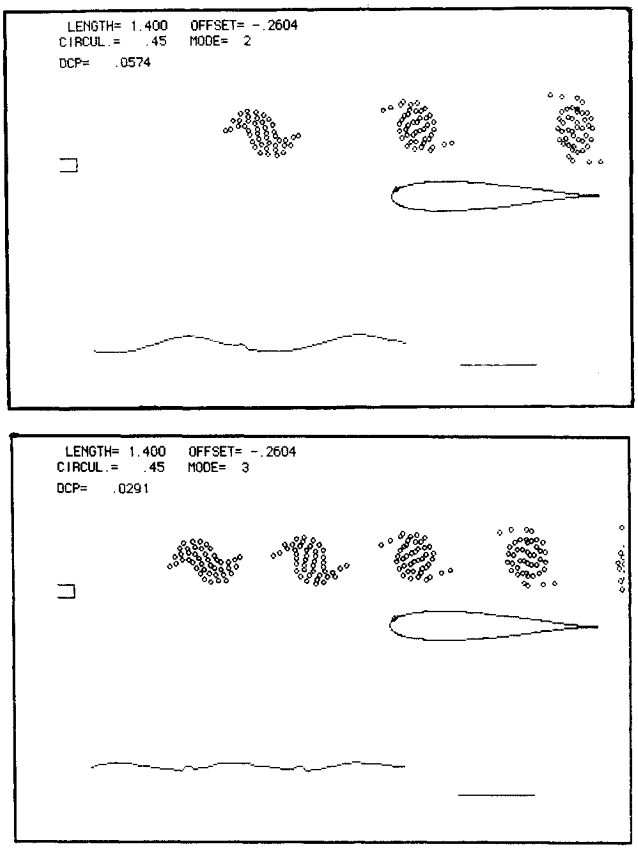

Fig. 9 Simulation of mixing layers passing above adjacent surfaces.

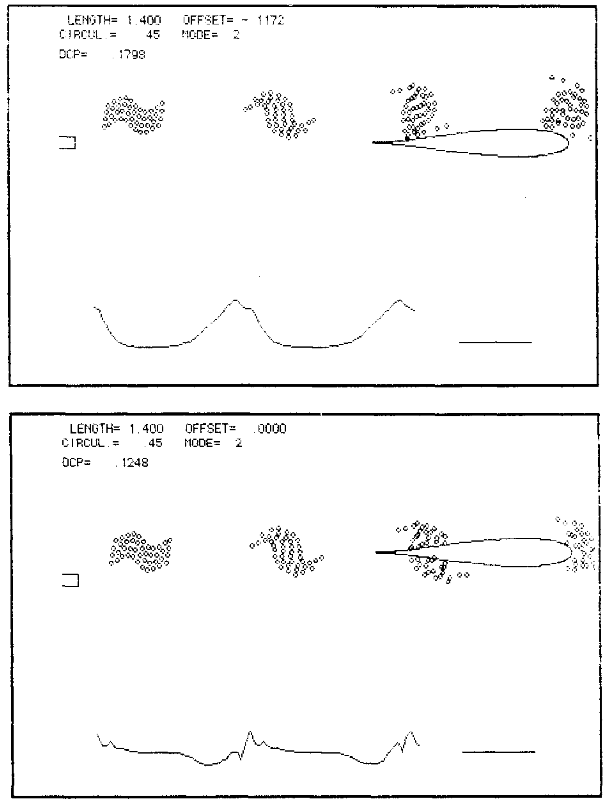

Fig. 10 Impingement on the sharp edge of the airfoil.

of the interaction of a shear layer with an edge or a corner. The stages of evolution of the vortices, from the initial state of a piece of distributed vorticity to the formation of rounded vortices that deform when they pass close to the interacting surface or are split when they impinge on it, are quite similar to the corresponding stages of evolution of the laboratory vortices observed in the real shear-layer/surface interactions. The time-dependent pressure fluctuations that are induced on a point of the interacting surface have the shape of smooth waves when the vortices do not come in contact with the surface or present subharmonics when the vortices impinge on the surface and are split.

A parametric application of the present method indicates that the amplitude of the pressure fluctuations depends strongly on the length of the succession of the vortices upstream of the edge and on the frequency of emission of the vortices. This amplitude becomes smaller when the spacing of the vortices is decreased while the length remains constant, or when the length is decreased and the mode of emission of the vortices ($n = L/\lambda$) is constant. Besides, as was mentioned in the introduction, there is strong evidence that, for the excitation of a mixing layer to the level required for the onset of self-sustained oscillations, the amplitude of the applied feedback force has to be greater than a threshold amplitude. These observations are useful in understanding the following features of a self-oscillating impinging shear-layer: when the length of the shear layer is constant, there is a specific limit of the maximum mode of oscillation that can be established, whereas, when the shear-layer length is increased, a critical value is reached above which the next mode appears.

A close examination of the various cases presented in this paper shows that, though a vortex spacing λ conforming to the equation $n = L/\lambda$ has been assumed for their estimation, the actual periodic cycles m of the pressure induced on the edge during the motion of one specific vortex from the origin of the flow to the edge are less than the mode number n. The relation $m = L/\lambda - \epsilon$ is valid in any particular mode of oscillation. The

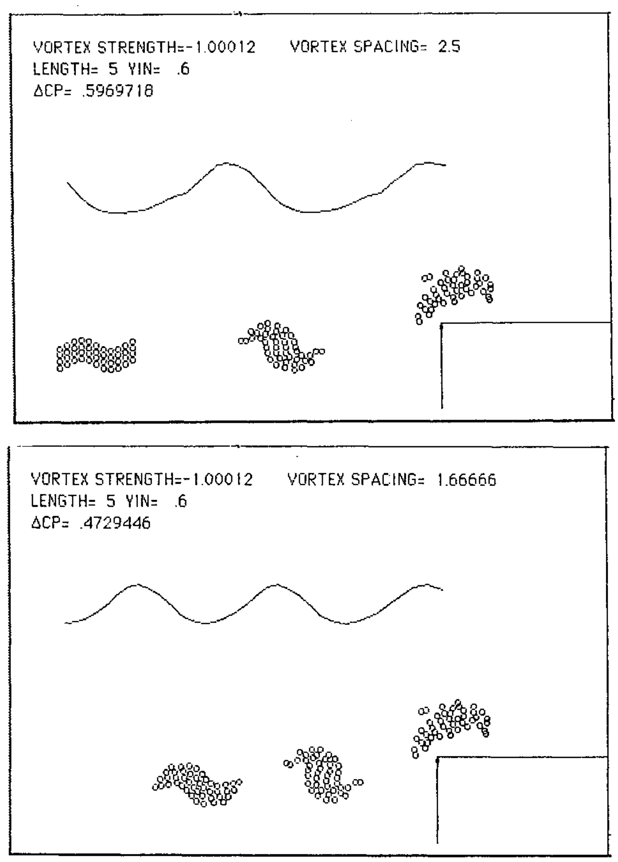

Fig. 11 Vortex/corner interactions, effect of the vortex spacing.

constant ϵ is approximately equal to 1/4. This relation is similar to the one found experimentally by Sarohia.[21] The difference between n and m is due to the variation of the convection velocity of the vortices along their trajectory. These

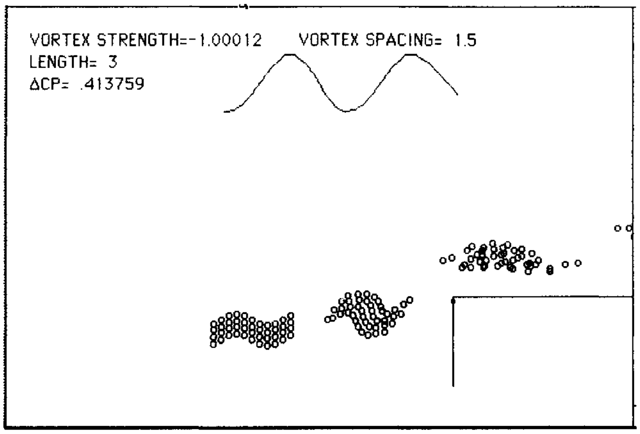

Fig. 12 Vortex/corner interactions, effect of the cavity length.

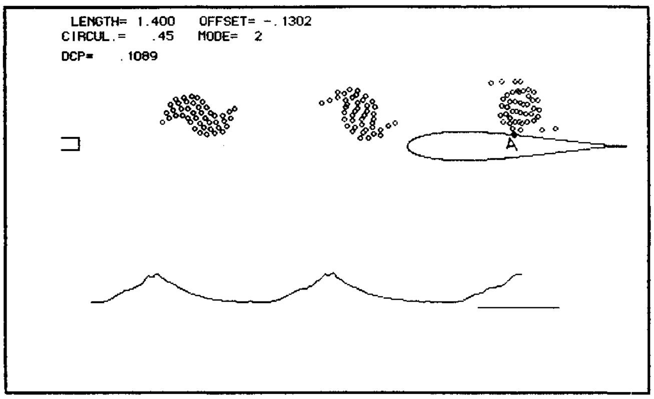

Fig. 13 Pressures fluctuations far from the leading edge.

results indicate that the spacing of the generated vortices in a self-oscillating impinging shear layer is probably not constant, but rather presents a small variation.

Also, according to the results of the parametric analysis performed in Sec. III, when a surface is placed near a succession of vortices, the amplitude of the pressure waves induced on the surface takes significant values only when the vortices pass very close to it. If the succession is displaced to a height equal only to the diameter of the vortices, the mean pressure remains almost constant, but the pressure amplitude decreases rapidly. This observation may be useful for understanding the role of reflectors and for evaluating the effect of a jet or of a mixing layer on an adjacent surface.

Concerning the leading-edge nature of the interaction phenomenon, we have mentioned already that Booth[19] has shown experimentally that significant perturbation pressures appear only over the first 20% of the surface of a symmetric airfoil. The present analysis gives similar results, and it helps to investigate the reason for this strange behavior. In Fig. 13, an example is shown of the estimation of the pressure at a point far from the leading edge. It is seen that, indeed, the amplitude of the fluctuations is very small. We note again that the calculated pressure fluctuations are composed of the steady and of the unsteady component, and that the unsteady component is opposite to the steady one. If the steady pressure results given by the present author in Ref. 14 are reviewed, it is seen that steady pressure fluctuations are induced all around the surface of the airfoil. It is evident then that in the present case, where both the components are considered, the steady pressure fluctuations have been offset by the unsteady component. It may be concluded that near the leading edge, where there are significant geometric variations, the steady pressure is higher than the unsteady contribution, and so there is a strong effect of the convecting vortices on the surface pressure. However, far from the leading edge, the two pressure components have similar amplitude, and the net effect is insignificant. It is noted, however, that in the case of highly curved airfoils, the steady-pressure maximum appears downstream of the leading edge; and so in this case the resultant pressure disturbance may be different than zero far from the leading edge at the region of maximum curvature.

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